Home Work in Data Reduction and Error Analysis

Chem 3111-Computers in Chemistry

October 3, 2002

Complete the following problems. Since this involves "pencil and paper", please turn in a hard copy to Prof. Bittner's mail box in the Chemistry office. If Mathematica is still working (i.e. I think there's a problem with the University's license), try to do problems using the notebook front end, using Mathematica to perform the symbolic manipulations.

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1. Find the relation between the uncertainty σ_x in x as a function of the uncertainties σ_u and σ_v for the following functions:

$$x = a\ln(\pm bu) \quad (1)$$
$$x = au^{\pm bv} \quad (2)$$
$$x = av \pm \frac{b}{u} \quad (3)$$
$$x = au\sin(vb) \quad (4)$$
$$x = a\exp(\pm bu^2) \quad (5)$$

2. If a table is round and its diameter is known to 1%, how well is its area known? Would it be better to measure the radius to 1% instead?

3. The electrical resistance, R, of a cylinder is proportional to its length, L, and inversely proportional to its cross-sectional area, $A = \pi r^2$. Which should be determined with higher precision, r or L, to optimize the determination of R? How much higher?

4. *Error in experimental design:* In designing an experiment to measure the Rydberg constant, R_y, using the spectral transitions of atomic Hydrogen, is it more accurate to use the higher energy photon from the 1s-2p transition from the Lyman series or the lower energy photon 2s-3p transition from the Balmer series? If your photon energy measurement is 1 part in 10^6, how accurate is your measurement of R_y?

5. *Error estimation in the Mullikan oil drop experiment:* In 1909, Robert Mullikan at the Univ. of Chicago provided a definitive measurement of the charge of an electron, e, thereby establishing the electron as a particle with a given charge and mass. In the experiment, small drops of oil were sprayed via an atomizer into a vessel and allowed to drift into a region between two parallel charged condenser plates. Although the drops were very small (radius of about a few microns) they could be illuminated and observed using a small optical telescope. When x-rays

are passed through the vessel, they partially ionized the air and some of the ejected electrons are captured by the oil droplets–that then become negatively charged and experience a force due the the electric field.

The first part of the experiment is performed with the field turned off, so that a particle of mass M falls under the gravitational force Mg. Since air is present in the vessel, the particle accelerates until it reaches its terminal velocity where the viscosity of air balances the gravitational force. This we determine via Stokes' law where by a spherical object of radius r moving with velocity v through a viscous medium with viscosity η experiences a frictional force

$$f = 6\pi r\eta v.$$

The forces are balanced when

$$Mg = 6\pi r\eta v_1.$$

The constant velocity v_1 can be measured by recording the time for a single drop to traverse two cross hairs in the telescope. The effective mass of the drop, corrected for the bouyancy of air) is related to the radius of the drop:

$$M = \frac{4}{3}\pi r^3(\sigma - \rho)$$

where σ and ρ are the densities of the drop and air respectively. Solving for r,

$$r = \left(\frac{9\eta v_1}{2g(\sigma - \rho)}\right)^{1/2}.$$

In the second part, the electric field ($\mathcal{E}$) is turned on so that the velocity of the drop is slowed or even reversed, depending upon the size and charge. When the new terminal velocity is reached, all the various forces are balanced by

$$\frac{4}{3}\pi r^3(\sigma - \rho)g = 6\pi r\eta v_2 + ne\mathcal{E}$$

Solving for ne,

$$ne = \frac{36\pi}{\mathcal{E}}\left(\frac{\eta}{2}\right)^{3/2}\frac{v_1^{1/2}(v_1 - v_2)}{(g(\sigma - \rho))^{1/2}}.$$

Consequently, knowing v_1 and v_2, Mullikan could determine ne. Since there was no way for him to know how many electrons had attached to a single drop, Mullikan repeated the experiment a number of times for a large number of drops and for different charges on the same drop. The greatest common divisor from the resulting values of ne was taken as the electronic charge e.

Down load the data file Mullikan.dat containing a portion of Mullikan's raw data for the fall times, t_1 and rise times t_2 in the gravitational field + electric field for a single oil drop. (These give you v_1 and v_2.) Using a spread-sheet (such as Excel) or Mathematica, calculate ne for each experiment and determine the greatest common divisor, e. Estimate the probable error and rms deviation of your result.

Some experimental parameters:

- plate distance = 16 mm
- fall distance = 10.21 mm

- electrical potential 16.96 statvolts (note $\mathcal{E}$ has units of statvolts/cm).
- air density = 1.1871 $\times 10^{-3}$g/cm^3
- oil density = 0.9199 g/cm^3
- air viscosity = 1.824 $\times 10^{-4}$ poise